APPENDIX I

S-N Formula

In this Appendix, we illustrate how the *S-N* curves such as the one in Figure 8.22 can be interpreted mathematically, and how problems such as Sample Problem 8.1 can be solved using the generalized *S-N* formula.

I.1 S-N Formula

First of all, observe that, in the *S-N* curves (for example, see Figure 8.22), one plots the *logarithm of the strength* on the *y*-axis, while on the *x*-axis, one plots the logarithm of the number of cycles (*N*). Since the plots are linear, we have:

$$\log_{10}(S) = A\log_{10}(N) + B \tag{I.1}$$

The two constants *A* and *B* are unknown, and can be determined as follows. Let the strength at $N = 10^6$ be $S = S_n$ (endurance limit), let the strength at $N = 10^3$ be $S = S_3$. Substituting these values in Equation (I.1), we obtain:

$$\begin{aligned}\log_{10}(S_n) &= A\log_{10}(10^6) + B\\ \log_{10}(S_3) &= A\log_{10}(10^3) + B\end{aligned}$$

Recall that $\log_{10}(10^m) = m$. Thus:

$$\begin{aligned}\log_{10}(S_n) &= 6A + B\\ \log_{10}(S_3) &= 3A + B\end{aligned}$$

Solving for *A* and *B*:

$$A = \frac{1}{3}\log_{10}\left(\frac{S_n}{S_3}\right); \quad B = \log_{10}\left(\frac{S_3^2}{S_n}\right)$$

Thus the general *S-N* formula is given by:

$$\log_{10}(S) = \frac{1}{3}\log_{10}\left(\frac{S_n}{S_3}\right)\log_{10}(N) + \log_{10}\left(\frac{S_3^2}{S_n}\right) \tag{I.2}$$

Now, given any value of *N*, one can find the corresponding strength, and vice versa.

I.2 Illustrative Example

Consider now Sample Problem 8.1, where it is given that $S_n = 61$ ksi and $S_3 = 112$ ksi. The objective is to find the strength corresponding to $N = 10^4$ cycles. From Equation (I.2), we have:

$$\log_{10}(S) = \frac{1}{3}\log_{10}\left(\frac{61}{112}\right)\log_{10}(N) + \log_{10}\left(\frac{112^2}{61}\right)$$

i.e.,

$$\log_{10}(S) = \frac{(-0.2639)}{3}\log_{10}(N) + 2.3131$$

For $N = 10^4$ cycles, we have:

$$\log_{10}(S) = \frac{(-0.2639)4}{3} + 2.3131 = 1.9612$$

Thus, at $N = 10^4$ cycles, $S = 10^{1.9612} = 91.45$ ksi.